# NONLINEAR CYCLIC TRUSS MODEL FOR SHEAR-CRITICAL REINFORCED CONCRETE COLUMNS

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**ABSTRACT:**

This study presents a nonlinear truss modeling approach for shear-critical reinforced concrete columns subjected to cyclic loading. Nonlinear steel and concrete truss elements are used to represent steel reinforcement and concrete areas, respectively, in the vertical and horizontal directions. Nonlinear concrete trusses are used in the diagonals, accounting for the biaxial effect on the compression behavior. Tension stiffening and softening effects are modeled for all concrete truss elements, accounting for mesh size effects and fracture energy, and the effects of strain penetration are modeled. Flexure-shear interaction is modeled explicitly through the coupling of these elements. The model is validated by comparing experimentally measured and numerically computed response for two shear-critical column specimens, both characterized by significant flexure-shear interaction effects and softening of the concrete in the diagonal direction. The overall force-deformation response is presented including significant strength degradation. In addition, the effects of the diagonal truss angle and concrete biaxial relationship on the response are studied.

**KEYWORDS:** Nonlinear, truss model, cyclic, shear-critical columns, flexure-shear interaction.

## 1. INTRODUCTION

There has been a great deal of experimental research on seismic behavior reinforced concrete columns subjected to cyclic reversals for decades. Reinforced concrete columns have been reported to have shear failures and flexure-shear failures especially in non –seismically designed RC structures during previous earthquakes. Flexure-shear interaction occurs as a result of coupling of nonlinearities due to axial, flexural, and shear forces. The computation of nonlinear cyclic response of reinforced concrete columns with flexure-shear interaction is still a challenging problem.

Modeling approaches for RC columns considering flexure-shear interaction for reinforced concrete columns can be categorized as (i) lumped plasticity models, (ii) fiber - section beam - column element models, (iii) macro models, (iv) truss or strut-and-tie models.

Lumped plasticity models use nonlinear hysteretic rules for shear and flexure springs located at the element ends to account for nonlinear flexure-shear interaction behavior (Lee and Elnashai 2001, Xu and Zhang 2011).

Fiber element models have been used extensively since 1970s. Mostafaei and Vecchio (2008) developed a uniaxial shear – flexure model for an element under monotonic loading considering average axial strains and average concrete compression strains. Ceresa et al. (2009) presented a two – dimensional Timoshenko fiber beam-column element and implemented to develop a flexure-shear model for RC beam – column (frame) elements under cyclic loading.

Macro model presented by Mergos and Kappos (2008) considered shear strength degradation of a beam-column element with two distributed flexibility sub-elements coupling considering shear and flexure deformations.

Lodhi and Sezen (2012) proposed a procedure with two macro models for flexure and shear deformations combined due to the dominant failure mode.

Truss or strut-and-tie models have been proposed in the literature to capture the strength and stiffness characteristics of RC members during cyclic reversals. Kim and Mander (1999) improved a truss model for monotonic and cyclic behavior of RC columns. Miki and Niwa (2004) proposed a three dimensional lattice model for biaxial responses of RC members. They used truss members and arch members spanning from top to bottom of the columns to represent shear resisting mechanism of RC columns. Park and Eom (2007) improved a truss model consisting composite elements of concrete and rebar for RC members. These models didn't account for the mesh size effects for concrete material models. Zimmerman et al. (2013) used nonlinear truss model for numerical modeling for shear failure of RC columns with considering strain penetration effects. This model didn't consider biaxial effect for concrete diagonals in compression.

Panagiotou et al. (2011) presented a nonlinear cyclic truss modeling approach including flexure-shear interaction (FSI) for plane stress RC members subjected to cyclic loading. Lu and Panagiotou (2013) developed a nonlinear beam – truss modeling approach considering flexure-shear interaction for non-planar RC walls subjected to uni-axial or multi-axial loading. These models accounted for biaxial effect for concrete diagonals in compression, mesh size effects for concrete and used a parallel angle model.

This study presents a nonlinear truss modeling approach for shear-critical reinforced concrete columns subjected to cyclic loading. Nonlinear concrete truss elements accounts for tension softening for the biaxial effect on the compression behavior in the diagonal direction and tension stiffening in the vertical and horizontal directions developed by Lu and Panagiotou (2013) are used. In addition, this model considers strain penetration effects resulted by longitudinal reinforcement slip from anchorage of column to foundation. Flexure-shear interaction is modeled explicitly through the coupling of these elements. The model is validated by comparing measured and computed responses of two RC columns tested and both characterized by significant flexure-shear interaction effects.

## 2. NONLINEAR TRUSS MODELING APPROACH

The truss modeling approach in this paper uses nonlinear truss elements in the vertical, horizontal and diagonal directions. Nonlinear concrete truss elements accounts for tension softening for the biaxial effect on the compression behavior in the diagonal direction and tension stiffening in the vertical and horizontal directions are used. The RC column section considered in non –seismically designed RC structures is shown in Figure 1(a). The clear height of the column is H. Column section width and height is $B_c$ and $H_c$, respectively. Figure 1(b) shows the truss model of the column. Vertical and horizontal truss elements representing reinforcing bars and concrete, and their effective areas are depicted in Figure 1 (c-d) referring to sub-segment A. The diagonal truss elements representing concrete only are shown in Figure 1 (e). The cross –section area of the diagonal truss elements is the product of the section width $B_c$ and effective width $b_{eff}$ of the column. Strain penetration effects due to anchorage deformations are considered using truss elements for reinforcement and concrete with a length of $L_{sp}=16d_{bl}$ is assumed for all case studies, in which $d_{bl}$ is the longitudinal bar diameter (Figure 1b). Strain penetration concrete truss elements have larger areas than the vertical concrete truss elements to account for the stiffness of the beams, while the area of steel truss elements are same with the vertical steel trusses. The diagonal truss angle $\theta_d$ ranged between $42^0$ and $52^0$ in the case studies and the effect of $\theta_d$ is considered in the Discussion section.

## 3. CONSTITUTIVE STRESS-STRAIN RELATIONSHIPS

### *3.1. Reinforcing Steel Material Model*

A number of Giuffré-Menegotto-Pinto (GMP) steel material models are used to define the stress-strain relationship used for the reinforcing steel.A single GMP model is shown in Figure 2, where $f_y$ is the yield strength, $\varepsilon_y$ the corresponding yield strain, $E_s$ the elastic modulus, and $B_s$ the post-yield hardening ratio – the monotonic envelope for this material model is bilinear and a multi-linear envelope is readily achieved by a parallel model. For the case studies presented in this paper, five to six GMP models are used in each parallel steel material model and the material parameters are chosen to match the experimentally reported steel behavior. The parameters controlling the transition between elastic and plastic branches ($R_0$, $cR_1$, $cR_2$) as defined in Filippou et al. (1983), are: $R_0$ =15, $cR_1$ = 0.9, $cR_2$ = 0.15 for all case studies. The effect of rebar buckling is not considered.

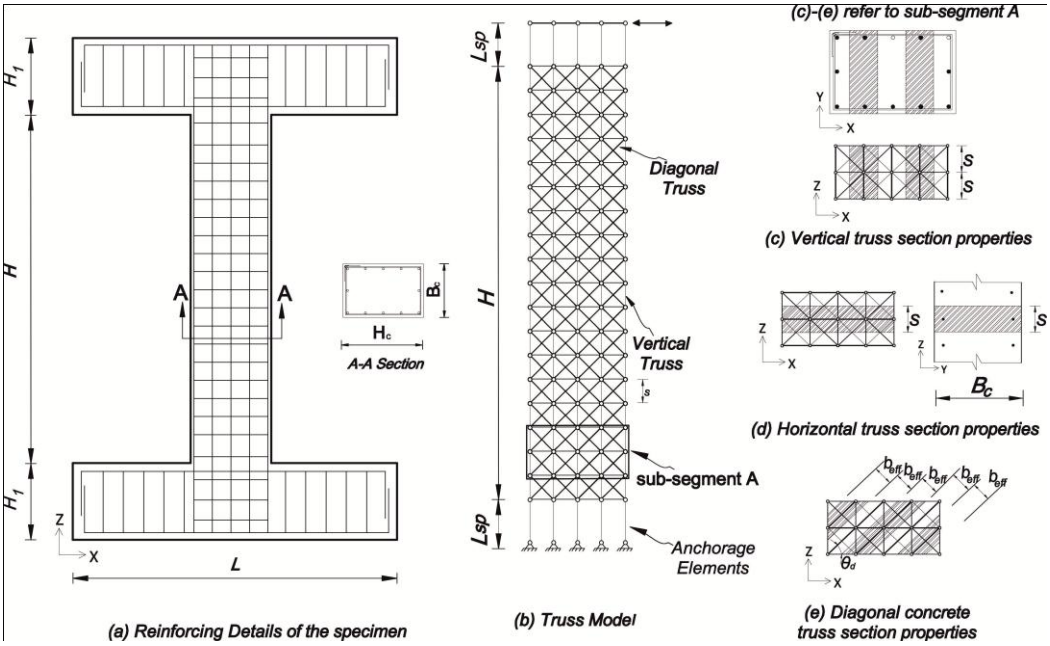



Figure 1. Nonlinear truss modeling approach for a RC column specimen

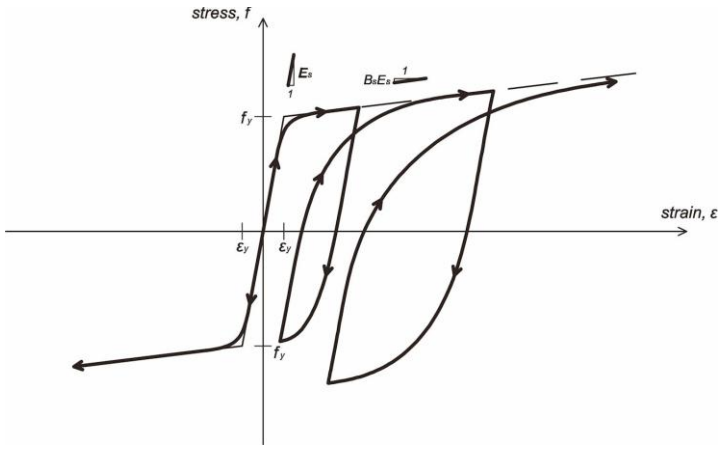



Figure 2. (a) Stress-strain relationship of the GMP steel material model

### *3.2. Concrete Model for Vertical and Horizontal Truss Elements*

The compressive stress-strain relation is based on the Fujii concrete model (Hoshikuma et al. 1997). The stress-strain relation for concrete developed by Lu and Panagiotou (2013) is used for the models and shown in Figure 3, where $f_c'$ is the compressive strength of unconfined concrete occurring at strain $\varepsilon_0 = 0.2\%$. The initial concrete modulus was $E_c = 2 f_c' / \varepsilon_0$. The value of $\varepsilon_u$ accounted for mesh-size effects base on the notion of concrete fracture energy in compression (Bazant and Planas 1998). The reference length ($L_R$) based on the panel studies in Vecchio and Collins (1986) is taken and for $L = L_R = 600\,\text{mm}$, $\varepsilon_u = 0.4\%$.

The tension stress-strain relationship during loading is linear until it reaches the tensile strength of concrete $f_t = 0.33\sqrt{f_c'}$ in MPa. After this point, the concrete softens in accordance to the tension stiffening equation of Stevens et al. (1991), which has parameters for the bar diameter and steel ratio in the direction of the bar. Upon unloading from a compressive strain, the tangent modulus is $E_u = 0.5 \cdot E_c + 0.5 \cdot (f / \varepsilon)$ until reaching zero stress, which then reloads linearly to the point with the largest tensile strain that occurred before. The unloading from a tensile strain is linear with a tangent modulus $E_c$ until reaching zero stress. After this, the material loads in compression and targets a stress equal to $\alpha \cdot f_t$ at zero strain with $\alpha = 0.5$.

### *3.3. Concrete Model for Diagonal Truss Elements*

The concrete material model used for the diagonal truss elements accounts for the bi-axial strain field on the concrete compressive behavior as described by Vecchio and Collins (1986). For truss element $e_1$ extending from node 1 to node 2 [shown in Figure 4(a)], the normal strain, $\varepsilon_n$, is computed using the zero-stiffness gauge element extending from the mid-length of the element to nodes 3 and 4, $g_1$ and $g_2$, respectively. The instantaneous compressive stress of element $e_1$ is multiplied by the factor $\beta$ determined from the instantaneous normal strain $\varepsilon_n$, which is the average of the strain measured with the gauge elements $g_1$ and $g_2$. When $\varepsilon_n > 0$, the relationship between $\beta$ and $\varepsilon_n$ is tri-linear, as shown in Figure 4(b).

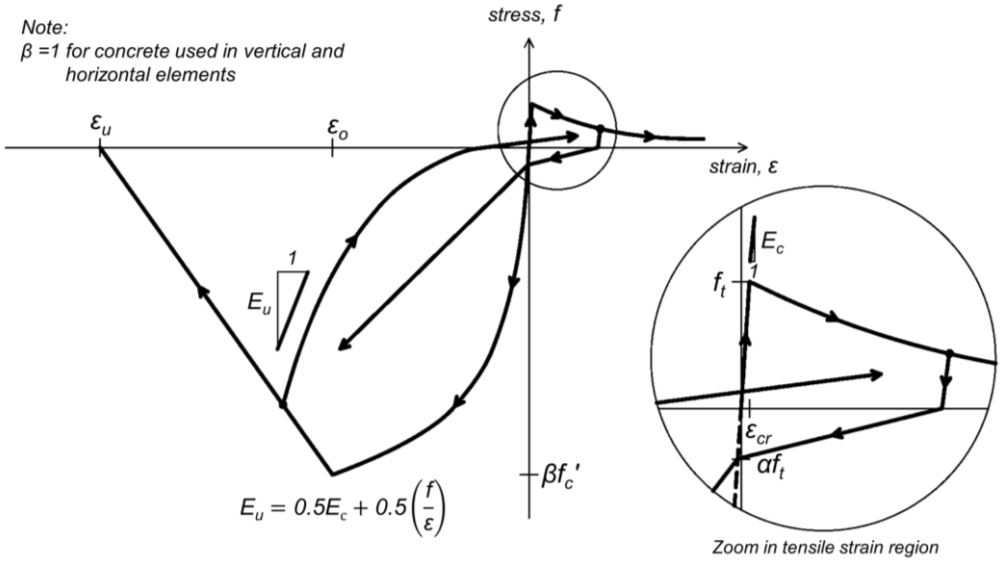



Figure 3. Stress-strain relationship for concrete material models

For this study, the relation between β and $\varepsilon_n$ depends on the length of the gauge elements, as first proposed by Panagiotou et al. (2013). Here, $\varepsilon_{\text{int}} = (600 / L_g) \cdot 1\%$ and $\varepsilon_{res} = (600 / L_g) \cdot 2.5\%$, where $L_g$ is the total length of gauge elements $g_1$ and $g_2$, as shown in Figure 4(a). The value of $\beta_{\text{int}} = 0.3$ and $\beta_{\text{res}} = 0.1$ was chosen to be similar to that developed by Vecchio and Collins (1986).

Concrete model for diagonals in tension has $f_{t,res} = 0.02\, f_t$ at tensile strain $\varepsilon_{t,res}$. Element length effect is accounted for $\varepsilon_{t,res}$, and $\varepsilon_{t,res} = 75\varepsilon_{cr}\,(L_R / L_D)$ as a function of diagonal element length ($L_D$) for $L_R = 600$ mm.

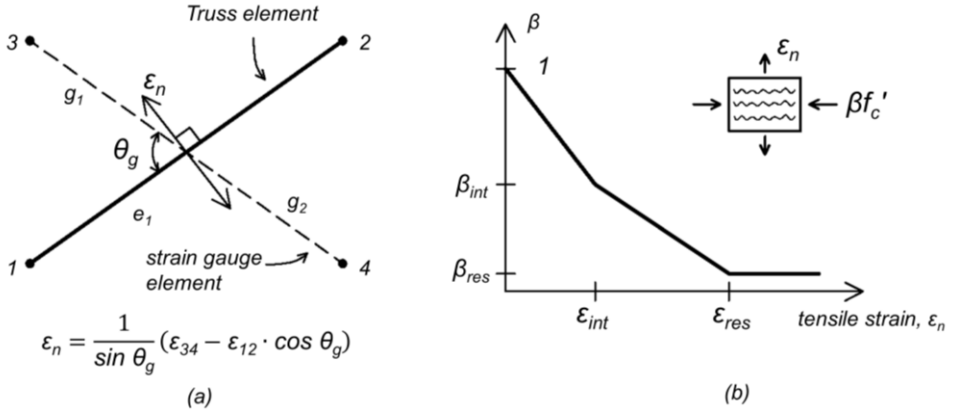



Figure 4. (a) Truss element accounting for biaxial effects in the compressive stress-strain behavior of concrete. (b) Relation between concrete compressive stress reduction factor, β, and normal strain, $\varepsilon_n$.

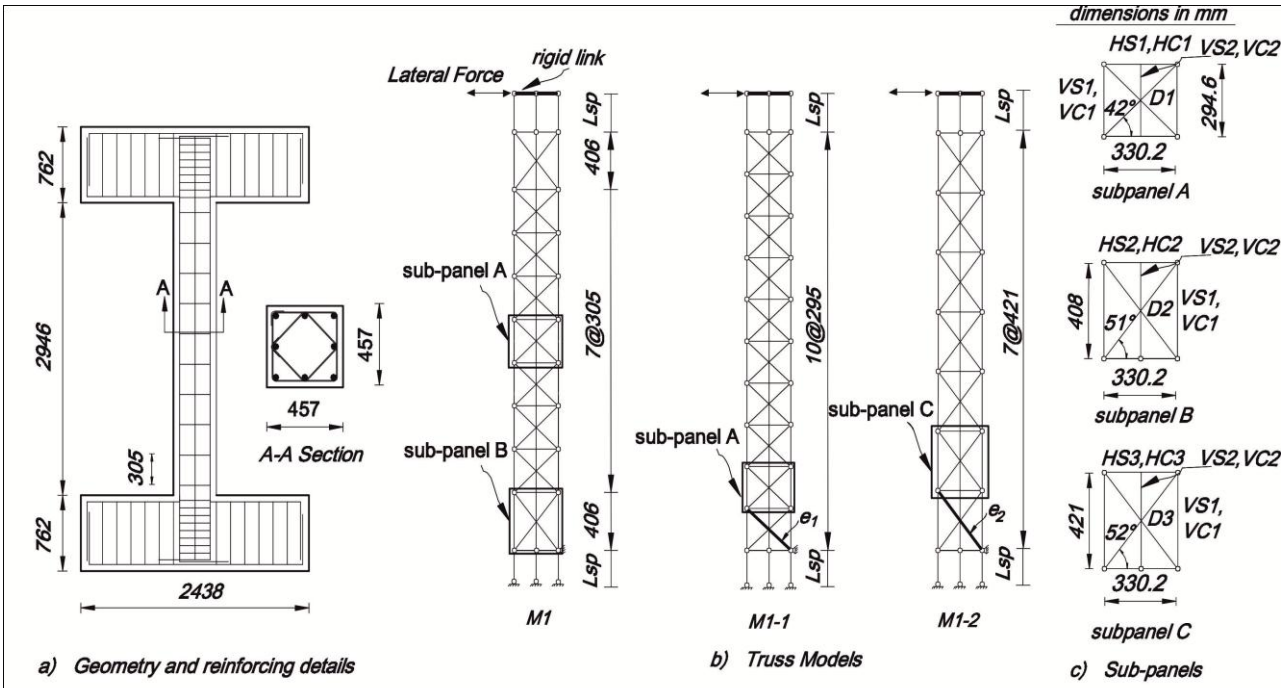
## 4. MODEL VALIDATION

The computer program OpenSees (McKenna et al. 2000) was used for all the case studies in this paper. The existing parallel material and GMP steel model, Steel02, was used for the parallel steel model. Both the uni-axial and bi-axial concrete models as well as the four-node truss element used in the diagonals, as described above, were implemented in Opensees by Lu and Panagiotou (2013). The response was validated with the test data of two columns and computed using a displacement control algorithm with uniaxial loading.

### *4.1. Case Study 1 - Sezen & Moehle (2006)*

In this case study, a square section RC column called Specimen 1 was considered (Figure 5). The longitudinal and the volumetric transverse reinforcement ratio was $\rho_l = 3.7\%$ and $\rho_t = 0.17\%$, respectively. Concrete and steel material properties are listed in Figure 5. Aspect ratio ($a / d$) of the column was 3.4. Constant axial load was applied to column specimen during the cyclic load reversals. The axial load ratio of the column $N / f_c' A_g = 0.15$ where N is the vertical load applied to the section and $A_g$ is the gross section area. Lateral displacements were applied to the specimen in terms of nominal yield displacement ($\Delta_y$) as $\Delta_y / 4, \Delta_y / 2, 2\Delta_y$, and $3\Delta_y$, until failure, with three cycles at each amplitude. The drift ratio Θ is defined as $\Delta / H$, where Δ is the lateral displacement and H is the height of the column where the load is applied. Sezen and Moehle (2006) reported that for Specimen 1, after spalling of cover concrete at 1.8 % drift ratio (first cycle), diagonal cracks at the mid-height of the column took place at the subsequent cycle and strength degradation started during the experimental study. After 2.7% drift ratio, the strength degraded nearly to zero while carrying the axial load.

M1 truss model was developed considering actual positions of the stirrups for Specimen 1 is given in Figure 5 (b-c). Vertical, horizontal and diagonal truss element areas are given in the table. The lateral force–displacement responses for experimentally measured and cyclically and monotonically computed using M1 model are compared in Figure 7(a). The failure mode observed in the case study is captured with the diagonal concrete crushing. The peak strength in Model M1 is 1.08 times the experimentally measured, and the failure happened at 1.8 % drift ratio. The effect of diagonal truss is presented in the Discussion section.




*Material Properties*

*Reinforcing Steel:*

| | #3 (9.5 mm) | #9 (28.65 mm) |
|---|---|---|
| $f_y$ (MPa) | 476 | 438 |
| $f_u$(MPa) | 724 | 645 |
| $\varepsilon_y$ | 0.18 % | 0.19 % |
| $\varepsilon_{sh}$ | 1.0 % | 1.3% |
| $\varepsilon_{ult}$ | 10% | 15% |

*Concrete:*

| | |
|---|---|
| $f_c'$ (MPa) | 21.1 |
| $\varepsilon_0$ | 0.2% |
| $f_t$ (MPa) | 1.52 |

*Vertical and Horizontal Truss Element Areas (mm²)*

| Concrete Areas: | | Steel Areas: | |
|---|---|---|---|
| $A_{VC1}$ | 52913 | $A_{VS1}$ | 1935 |
| $A_{VC2}$ | 104516 | $A_{VS2}$ | 1290 |
| $A_{HC1}$ | 97277 | $A_{HS1}$ | 242 |
| $A_{HC2}$ | 134722 | $A_{HS2}$ | 269 |
| $A_{HC3}$ | 139014 | $A_{HS3}$ | 311 |

*Diagonal Truss Element Areas (mm²)*

| | |
|---|---|
| $A_{D1}$ | 100095 |
| $A_{D2}$ | 117392 |
| $A_{D3}$ | 118503 |

Figure 5. Case Study 1 – Sezen & Moehle (2006) – Specimen 1

### *4.2. Case Study 2 – Priestley et al. (1994)*

Case study 2 is a rectangular RC column, called R1A, with an aspect ratio $M/VD=2.0$, see Figure 6 (a). The longitudinal and the volumetric transverse reinforcement ratio was $\rho_l=2.52\%$ and $\rho_t=0.083\%$, respectively. Concrete and steel material properties are listed in Figure 6. Constant axial load was applied to column specimen during the cyclic load reversals. The axial load ratio of the column $N/f_c'A_g=0.054$ where N is the vertical load applied to the section and $A_g$ is the gross section area. The column was subjected to and initial force – controlled stage, following a displacement controlled loading pattern in terms of displacement ductility factors. The drift ratio Θ is defined as $\Delta/H$, where Δ is the lateral displacement and H is the height of the column where the load is applied. R1A specimen showed limited ductile response until the initial cycles at 0.8% drift ratio.

At third cycles of 1.4 % drift ratio rapid strength degradation started with a major diagonal crack in the lower portion of the column (Priestley et al., 1994).

M2 truss model was developed due to actual positions of the stirrups for R1A test unit is given in Figure 6 (b-c). Vertical, horizontal and diagonal truss element areas are given in the table. The lateral force – lateral displacement responses for experimentally measured and cyclically and monotonically computed using M2 model are compared in Figure 7(b). The failure mode observed in the case study is captured with the diagonal concrete crushing. The peak strength in Model M2 is 1.06 times the experimentally measured, and the failure happened at 0.8 % drift ratio.

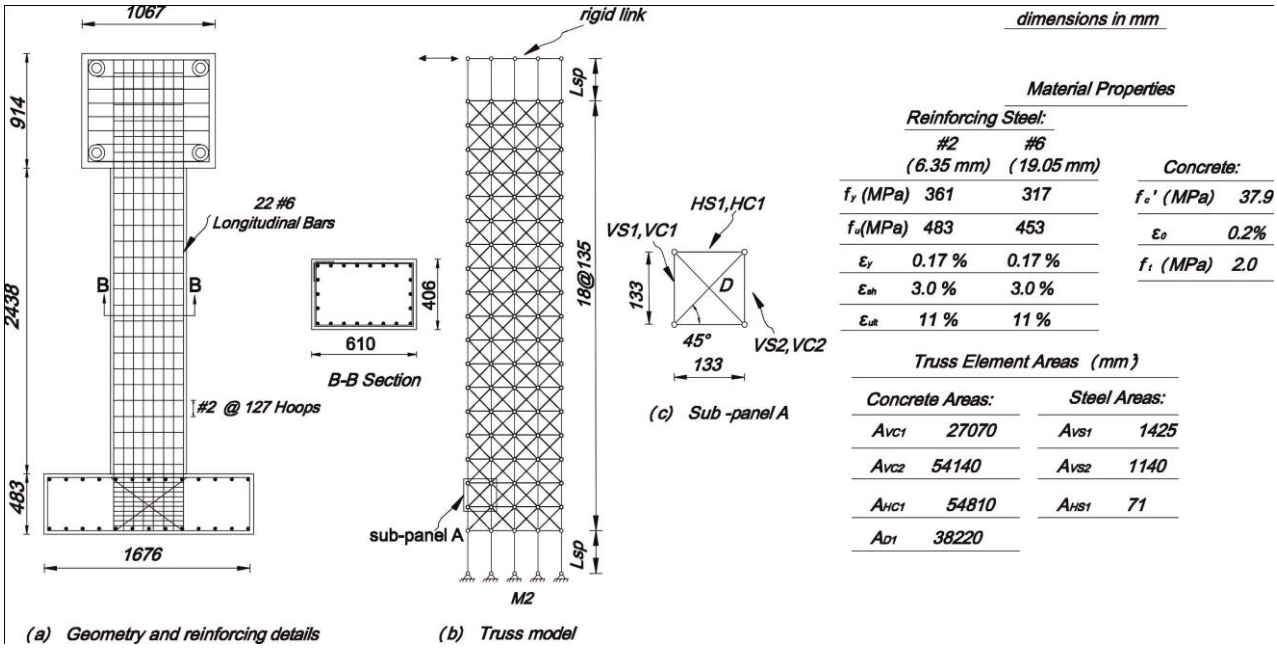



Figure 6. Case Study 2 – Priestley et al (1994) – R1A test unit.

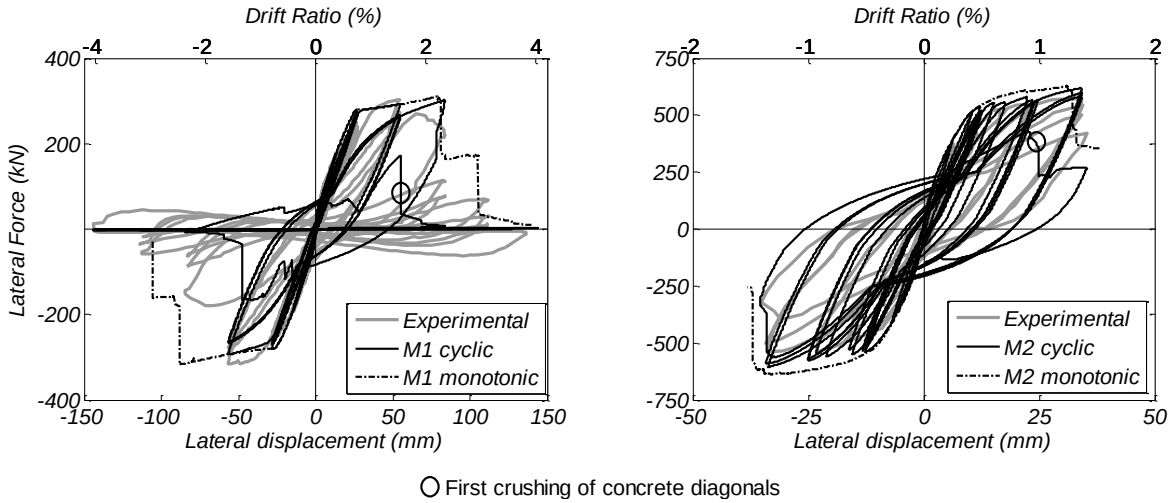



Figure 7. Comparison of experimentally measured and computed responses for (a) Case Study 1 and (b) Case Study 2.

## 5. DISCUSSION

It is shown in the previous section that and numerically computed responses using nonlinear truss modeling approach give very satisfactory results in comparison with experimental data in case studies. In this section the effect of angle of inclination of diagonal truss elements on the response are studied.
In Case Study 1, M1-1 and M1-2 models have different diagonal truss angles about $42^0$ and $52^0$ are considered. For the truss models, diagonal elements $e_1$ and $e_2$ with initial crushing during analyses are shown in Figure 5(b). The computed cyclic and monotonic load- displacement responses for the two models are given in in Figure 8. The results show that as the diagonal truss angle increases, the initial crushing of diagonal concrete truss elements occur at smaller drift ratios (Figure 8c). For model M1-1 crushing of diagonal elements occur at 1.8 %, while for model M2-2 it happens at 2% drift ratio.

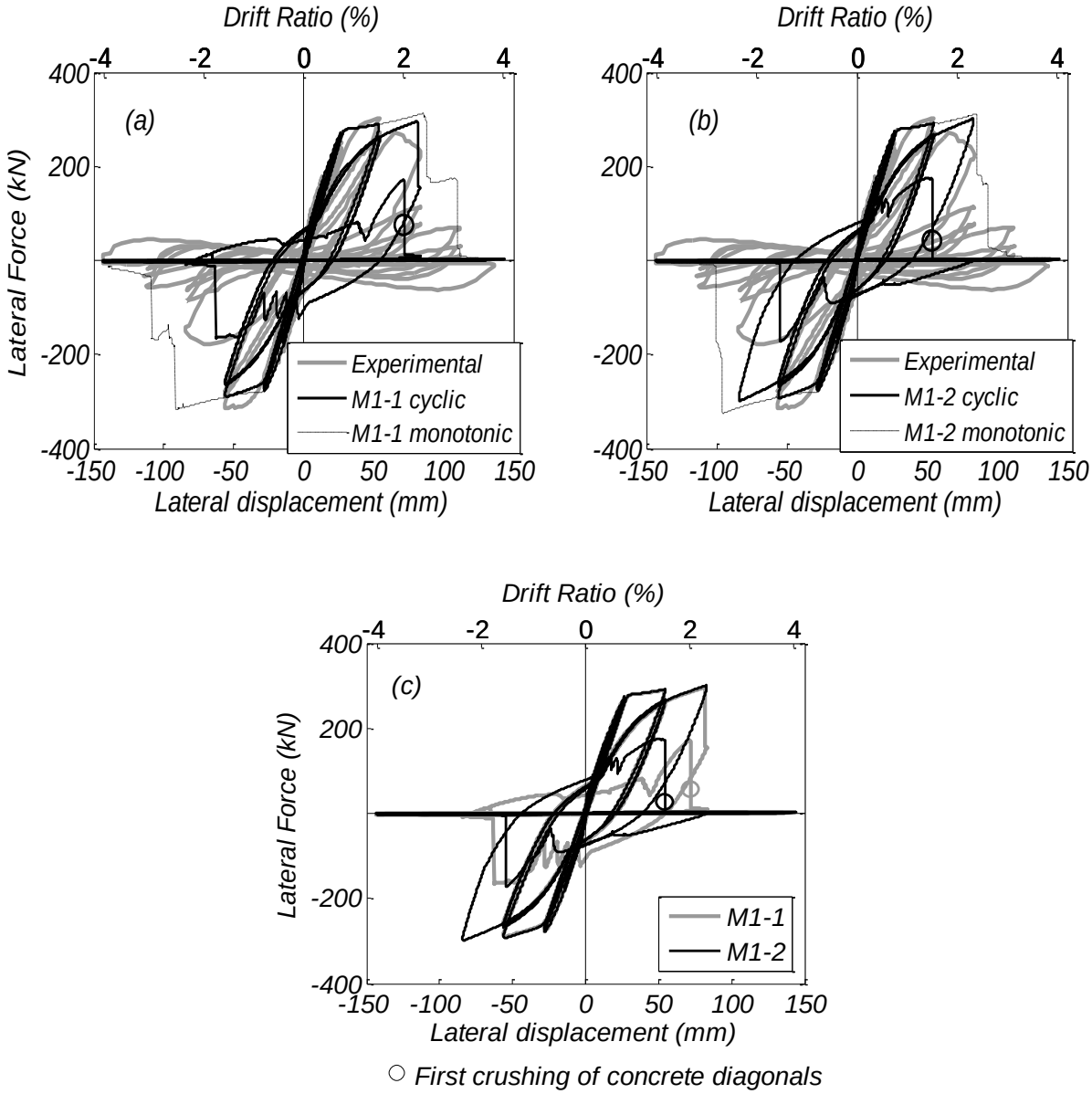



Figure 8. The comparison of response with different diagonal angles with experimental results for models (a) M1-1, (b) M1-2, (c) Effect of angle of diagonal truss elements for Case Study 1.

## 6. CONCLUSIONS

This paper describes a nonlinear truss modeling approach for RC columns suffering from failures resulted by flexure-shear interaction. The model accounts for the mesh size effects for concrete stress – strain relationship, biaxial effect on compression behavior for diagonal truss elements and strain penetration effects. The model is verified by comparison of computed and experimentally measured responses for two RC columns, both characterized by significant flexure-shear interaction effects and softening of the concrete in the diagonal direction. The main conclusions drawn in the study:

1. The nonlinear cyclic truss models computes satisfactorily the post-cracking cyclic force – displacement response and overall response of RC columns subjected to cyclic loading.
2. The model compute crushing of concrete diagonals for Case Study 1 and Case Study 2 in a good agreement with the test results.
3. The effect of angle of inclination for diagonal truss elements is investigated for Case Study 1. The results show that as the diagonal truss angle increases, the initial crushing of diagonal concrete truss elements occur at smaller drift ratios.

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