EXPERIMENTAL ESTIMATION OF TIME PERIOD OF VIBRATION FOR MOMENT RESISTING FRAME BUILDINGS

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ABSTRACT:

Most seismic design codes generally provide formulas to be used for the estimation of the base shear and lateral loads. For the determination of the lateral loads, it is required to estimate first the fundamental vibration period of the building theoretically or experimentally.
In various current codes and also in the recommendations of many researches, empirical formulas relate the building fundamental period of vibration (*T*) to the building overall height (*H*) or the number of stories (*N*) without consideration of the two factors together. In this paper, Based on regression analysis of the available data for the fundamental vibration period of reinforced concrete moment-resisting frame buildings, measured from their motions recorded during eight California earthquakes. Improved formulas for estimating the fundamental period of vibration (*T*) of reinforced concrete moment-resisting frame buildings are developed by taking the effect of both building overall length and number of stories together. The results indicate that the value of coefficient C_t in the current US and Egyptian building codes' formulas should be modified to be function of the number of stories (*N*). Comparisons between the periods determined using the proposed formula and the measured values show good agreement.

KEYWORDS: Moment resisting frame buildings, experimental estimation, vibration period, concrete frame, building design codes.

1. INTRODUCTION

The fundamental vibration period of a building appears in the equation specified in building codes to calculate the design base shear and lateral forces. Building design codes provide empirical formulas that depend on the building material [steel, reinforced concrete (RC), etc.], building type (frame, shear wall, etc.), and overall dimensions. The fundamental vibration period of buildings have a significant influence on the seismic induced lateral forces.
The period of formulas in the 1997 Uniform Building Code (UBC1997), the 1996 Structural Engineering Association of California (SEAOC) recommendation and the recent Egyptian Code (EGC2012) are derived from those developed in 1975 as part of the ATC3-06 project [Applied Technological Council (ATC) 1978] largely based on periods of buildings measured from their motions recorded during the 1971 San Fernando earthquake. Goel and Chopra 1997 [1] are developed improved empirical formulas to estimate the fundamental vibration period of RC moment resisting frames (MRF) buildings for use in equivalent lateral force analysis specified in building codes using motions of many buildings recorded during earthquakes. The data used in [1] have been combined from the motions of buildings recorded during the 1971 San Fernando, 1984 Morgan Hill, 1986 Mt. Lewis and Palm Spring, 1987 Whitter, 1989 Loma Prieta, 1990 Upland, 1991 Sierra Madre and 1994 Northridge earthquakes.
The objective of this paper is developing improved empirical formulas to estimate the fundamental vibration period of RC MRF buildings as a function in both the building overall height (*H*) and the number of stories (*N*) based on data given in [1].

2. PERIOD DATABASE

The data that used in the regression analysis at this paper are that used by Goel and Chopra (1997) data [1]. These available data on the fundamental period of buildings measured from their motions recorded during eight California earthquakes, starting with the 1971 San Fernando earthquake and ending with 1994 Northridge earthquake.
Table 1 shows the subset of this data base pertaining 37 data points for 27 RC MRF buildings.
The number of data points exceeds the number of buildings because the period of some buildings was determined from their motions recorded during more than one earthquake or was reported by more than one investigator for the same earthquake.
From these data, It can be noticed that the significant difference of the floor heights where it range between 8.45 - 23.8 ft which take our attention to the effect of the number of stories (*N*) in the fundamental period of vibration (*T*).

Table 1. Period Data for RC MRF Buildings

No (1)	Location (2)	ID number (3)	No. of stories (4)	Overall Height (ft) (5)	Floor Height (ft) (6)	Earthquake (7)	Period *T* (s)	
							Longitudinal (8)	Transverse (9)
1	Emeryville	NA	30	300.0	10	L.oma Prieta	2.80	2.80
2	Los Angeles	NA	9	120.0	13.33	San Fernando	1.40	1.30
3	Los Angeles	NA	14	160,0	11.43	San Fernando	1.80	1.60
4	Los Angeles	NA	13	166.0	12.77	San Fernando	1.90	2.40
5	Los Angeles	ATC_12	10	137.5	13.75	San Fernando	1.40	1.60
6	Los Angeles	ATC_14	7	61.0	8.714	San Fernando	0.90	1.20
7	Los Angeles	ATC_2	7	68.0	9.714	San Fernando	1.00	1.00
8	Los Angeles	ATC_3	12	159.0	13.25	San Fernando	SW	1.33
9	Los Angeles	ATC_5	19	196.8	10.36	San Fernando	2.15	2.22
10	Los Angeles	ATC_6	11	124.0	11.27	San Fernando	1.43	1.60
11	Los Angeles	ATC_7	22	204.3	9.286	San Fernando	1.90	2.20
12	Los Angeles	ATC_9	16	152.0	9.5	San Fernando	1.10	1.80
13*	Los Angeles	C24236	14	148.8	10.63	Northridge	NA	2.28
14*	Los Angeles	C24463	5	119.0	23.8	Northridge	1.46	1.61
15*	Los Angeles	C24463	5	119.0	23.8	Whittier	1.40	1.30
16*	Los Angeles	C24569	15	274.0	18.27	Northridge	3.11	3.19
17*	Los Angeles	C24579	9	141.0	15.67	Northridge	1.39	1.28
18*	Los Angeles	N220-2	20	196.8	9.84	San Fernando	2.27	2.09
19*	Los Angeles	N220-2	20	196.8	9.84	San Fernando	2.27	2.13
20*	Los Angeles	N220-2	20	196.8	9.84	San Fernando	2.24	1.98
21*	Los Angeles	N446-8	22	204.3	9.286	San Fernando	1.94	2.14
22*	Los Angeles	N446-8	22	204.3	9.286	San Fernando	1.84	2.17
23*	North Hollywood	C24464	20	169.0	8.45	Northridge	2.60	2.62
24	North Hollywood	C24464	20	169.0	8.45	Whittier	2.15	2.21
25	Pomona	C23511	2	30.0	15	Upland	0.28	0.30
26	Pomona	C23511	2	30.0	15	Whittier	0.27	0.29
27	San Eruno	C58490	6	78.0	13	Loma Prieta	0.85	1.10
28	San Bruno	C58490	6	78.0	13	Loma Prieta	0.85	1.02
29	San Jose	NA	5	65.0	13	Morgan Hill	0.83	0.83
30	San Jose	C57355	10	124.0	12.4	Loma Prieta	1.01	SW
31	San Jose	C57355	10	124.0	12.4	Morgan Hill	0.91	SW
32	San Jose	C57355	10	124.0	12.4	Mount Lewis	0.91	SW
33*	Sherman Oaks	ATC_4	13	124.0	9.538	San Fernando	1.20	1.40
34*	Sherman Oaks	C24322	13	184.5	14.19	Whittier	1.90	2.30
35*	Sherman Oaks	C24322	13	184.5	14.19	Whittier	NA	2.44
36	Van Nuys	ATC_l	7	65.7	9.386	San Fernando	0.79	0.88
37*	Van Nuys	C24386	7	65.7	9.386	Whittier	1.40	1.20

Note: *Denotes buildings with $\ddot{u}_{g0} \geq 0.15$; NA indicates data not available; SW implies shear walls form the lateral-load resisting system; number followed by "C" or "N" indicates the station number and by "ATC" indicates the building number in ATC3-06 report.

3. CODE FORMULAS

The empirical formulas for the fundamental vibration period of MRF buildings in most design codes such as U.S. building codes (UBC-97, ATC 1978, SEAOC-96, and NEHRP 1994) and the recent Egyptian code (EGC2012) are of the form

$$T = C_t H^{0.75} \tag{1}$$

Where H = overall height of the building in feet above the base; and C_t is a numerical coefficient related to the lateral-force-resisting system. The values of C_t specified in these codes are: 0.030 and 0.035 for RC and steel MRF buildings, respectively, with one exception: in ATC3-06 recommendations Ct = 0.025 for RC MRF buildings.
In some design codes such as the NEHRP-94 provisions and the earlier versions of other seismic codes as Egyptian code (EGC 1993), an alternative formula for RC MRF buildings

$$T = 0.1N \tag{2}$$

Where N = number of stories. The simple formula is restricted to buildings not exceeding 12 stories in height and having a minimum story height of 10 ft.
Most of current codes specify that the design base shear should be calculated from

$$V = C W \tag{3}$$

Where W = total seismic dead load; and C = seismic coefficient depended on the soil profile, seismic zone factor; important factor; The fundamental period T; and the numerical coefficient representative of the inherent over strength and global ductility capacity of the lateral-load-resisting system.
The fundamental period T, calculated using the empirical formulas in (1) and (2), should be smaller than the true period to obtain conservative estimate for base shear. Therefore, code formulas are calibrated internationally to underestimate the period by approximately 10-20% at first yield of the building.
The codes permit calculation of the period by a rational analysis, such as Rayleigh's method, but specify that the resulting value should not be longer than that estimated from the empirical formula by a certain factor.

4. EVALUATIONS OF CODE FORMULAS

In order to evaluate the code period formulas, the measured building period's records are compared with those obtained from the empirical code formula (Eq. 1) in Figure 1 where they are plotted against the building height (H).

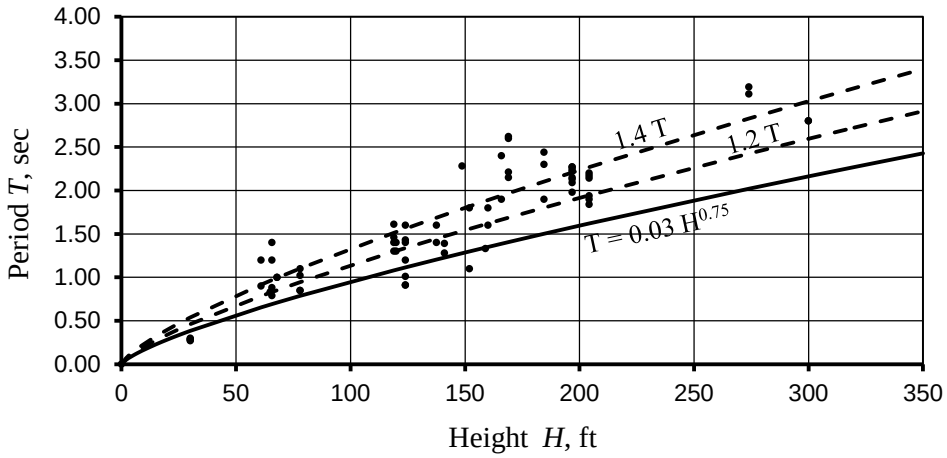

Figure 1. Comparison of measured and code periods for RC MRF buildings

The measured periods are shown by solid circles, whereas code periods are shown by curve denoted as *T*. Also, curves for 1.2*T* and 1.4*T* are included representing restrictions on the period from rational analysis imposed by various US and Egyptian codes.
From Figure 1 for all RC MRF buildings, we can observe the following

- The code formula leads to periods are generally shorter than measured periods.
- The code formula is close to the lower bound of measured periods for buildings up to 160 ft high.
- The code formula leads to periods significantly shorter than the measured periods for buildings in the range of 160 ft to 225 ft. the lower bound tends to be about 1.20times the code periods.
- Although for concrete MRF buildings taller than 225 ft the data is limited, the code formula would lead to much shorter period compared to measured periods for such buildings.

From the previous observations the coefficient C_t = 0.030 in current codes may be too conservative and should be modified to be function of the number of stories (*N*).

5. REGRESSION ANALYSIS METHOD

From the code formulas and recommended formulas in the recent researches, the suggested formula which is adopted in the present paper is of the form

$$T = \alpha N^{\beta} H^{\gamma} \tag{4}$$

In which constants α, β and γ depend on building properties, with γ bounded between one-half and one. This form is adopted in the present paper and constants α, β and γ are determined by regression analysis of the measured period data.
For the purpose of regression analysis, it is useful to recast (4) as

$$z = a + \beta x + \gamma y \tag{5}$$

In which $z = \log(T)$, $a = \log(\alpha)$, $x = \log(N)$, and $y = \log(H)$. The constants a, β and γ of the plane represented by (5) were determined by minimizing the squared error between the measured and computed periods, and then α was back-calculated from the relation $a = \log(\alpha)$. The standard error of estimate is

$$s_e = \sqrt{\frac{\sum_{i=1}^{n} [z_i - (a + \beta x_i + \gamma y_i)]^2}{(n-2)}} \tag{6}$$

Where $z_i = \log(T_i)$ = observed value (with T_i = measured period); ($a + \beta x_i + \gamma y_i$) = [$\log(\alpha) + \beta \log(N_i) + \gamma \log(H_i)$] = computed value of the *i*th data; and *n* = total number of data points. The s_e represents scatter in the data and approaches, for large *n*, the standard deviation of the measured periods from the best-fit equation.
This procedure leads to values of α, β and γ for (4) to represent the best-fit to the measured period data using least-squared method.
For code applications the formula should provide lower values of the period, and this was obtained by lowering the best-fit plane [see (5)] by s_e without changing its slopes. Thus α_L, the lower value of α, is computed from

$$\log(\alpha_L) = \log(\alpha) - s_e \tag{7}$$

This bound implies that only 15.9% of the measured periods would fall below the lower bound plane.
Also, codes specify an upper limit on the period calculated by rational analysis. This limit was obtained by raising the best-fit plane [see (5)] by s_e without changing its slopes. Thus α_U, the upper value of α, is computed from

$$\log(\alpha_U) = \log(\alpha) + s_e \tag{8}$$

This bound implies that only 15.9% of the measured periods would fall upper the lower bound plane.

6. RESULTS OF REGRESSION ANALYSIS

The theoretical form of Eq. (4) was adopted in the present investigation and constants α, β and γ were considered as variables. This unconstrained regression analysis led to the best possible fit and thus the minimum possible error between the measured and calculated periods (s_e =0.204). The obtained formula is

$$T = 0.027\, N^{0.17}\, H^{0.74} \tag{9}$$

In another regression analysis only α and β considered as variable; γ was fixed at 0.75 [rounded off to the nearest 0.05 of Eq. (9)] which concord to the most codes formulas. As expected, this constrained regression analysis led to negligible higher errors (s_e =0.204). The adjustment formula is

$$T = 0.026\, N^{0.16}\, H^{0.75} \tag{10}$$

The formula that is of interest for code-type application is the one that provides a lower bound to the measured data, denoted as T_L, correspondence to the best fit – 1 σ. The upper limit on the period calculated by rational analysis, denoted as T_U, correspondence to the best fit + 1 σ.
Eqs. (7) and (8) was used in the last regression analysis [Eq. (10)] to obtain α_L = 0.021 and α_U = 0.032 Leading to

$$T_L = 0.021 N^{0.16}\, H^{0.75} \tag{11}$$

and,

$$T_U = 0.032\, N^{0.16}\, H^{0.75} \tag{12}$$

If Eqs. (11) and (12) are plotted in 3D graph, very few data fall above the surface represented by T_U or below the surface represented by T_L. This indicates that (11) is suitable for estimating, conservatively, the fundamental period and (12) for limiting the period computed from rational analysis.

This period should not be longer than $1.5T_L$; the factor 1.50 is determined as the ratio 0.032:0.021, rounded-off to one digit after the decimal point.

Eq. (11) can be putted in the same form of the empirical formulas [Eq. (1)] for the fundamental vibration period of RC MRF buildings in most design codes where the coefficient C_t calculated as a function of the number of stories N as follows:

$$C_t = 0.021 N^{0.16} \tag{13}$$

Also, the coefficient C_t can be obtained using the relation curve given in Figure 2. According to this modified relation the coefficient C_t which equals to 0.030 in the most current codes will be range from 0.025 to 0.036 for number of stories N equals to 3 and 30 respectively.

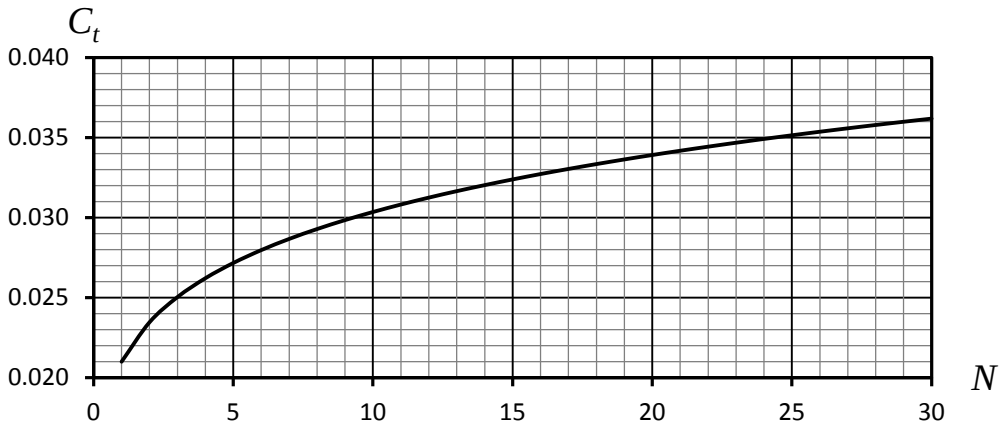

Figure 2. Coefficient C_t & N relation

7. COMPARISONS OF THE RESULTS

Using the data given in Table 1 for the fundamental vibration period of RC MRF buildings, Table (2) shows the comparison of the standard error obtained by Eq. (11) of the present work, and that suggested by Goel and Chopra [1] with that obtained by the empirical formula using in most design codes [Eq. (1)].
It can be noticed that the improved equation (11) gives the least standard error which may be used in the design codes.

Table 2. Comparison of results from regression analysis for RC MRF buildings

Regression Analysis Type	Period Formula		
	Best-fit	Best-fit - 1 σ	s_e
Most Current Codes Eq.(1)	$T= 0.038\ H^{0.75}$	$T_L=0.030\ H^{0.75}$	0.229
Goel and Chopra [1]	$T= 0.018\ H^{0.90}$	$T_L=0.015\ H^{0.90}$	0.211
Present Work Eqs. (10), (11)	$T= 0.026\ N^{0.16}\ H^{0.75}$	$T_L=0.021\ N^{0.16}\ H^{0.75}$	0.204

8. CONCLUSIONS AND RECOMMENDATIONS

For the determination of buildings periods of vibration of RC MRF buildings, it is necessary to take into consideration the effect of both the building overall length (*H*) and the number of stories (*N*) together. Improved formula for estimating the fundamental period of vibration (*T*) of RC MRF buildings are developed by regression analysis of the measured period data considering the coefficient C_t as a function of the number of stories *N*.

Based on analysis of the available data for the fundamental vibration period of 27 RC MRF buildings measured from their motions recorded during earthquakes, (11) is recommended for estimating the period of RC MRF buildings. This formula provides the best fit of (4) to the available data. The fit is better than possible equation (1) in most current codes. Furthermore, the period from rational analysis should not be allowed to be exceed the value from the recommended equation by a factor larger than 1.5.

Regression analysis that led to the recommended formula should be repeated periodically on larger data sets including buildings in other parts of the world where building design practice is significantly different than California.

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