H_2/LQG CONTROL OF SEISMIC VIBRATIONS OF CIVIL ENGINEERING STRUCTURES BY MAGNETORHEOLOGICAL DAMPERS

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ABSTRACT:

During the last thirty years designers have tend to utilize control systems in structures to construct extreme structures, to reduce afflicting losses in earthquakes, and to design buildings which comfortably withstand seismic events compared to conventional buildings. Nowadays there exist various types of control systems for civil engineering structures as passive, semi-active, active, and hybrid ones. Magnetorheological dampers are semi-active control systems which possess the advantages of both active and passive control devices. The damping properties of the MR dampers are regulated by different control strategies. The required feedback to control the system is established by potentiometers. Thus, only displacements are assumed to be sensed, and the required velocities of each story are not measured, as it is required for the full state information. Therefore the necessity of designing an observer becomes crucial in order to predict the unmeasurable states. For this reason, in the present research an H_2/LQG control strategy is utilized. The response of the control algorithm is compared with the LQR control strategy with full state feedback assumption. The three-storey model structure is subjected to an earthquake ground excitation in order to check the utilized control method. The results show that the H_2/LQG control reduces the structural responses by an acceptable ratio of approximately 40%, whereas some undesirable peaks in the damper force exist, which are unlikely to apply physically. There exist some increases in the first floor's absolute acceleration values. This case is an expected outcome due to the existence of the damper force in the first floor.

KEYWORDS : *MR damper, H_2/LQG control , Semi-active control.*

1. INTRODUCTION

The magnetorheological (MR) dampers are devices that utilize MR fluids which are suspensions of micron-sized, magnetizable particles randomly dispersed in a carrier medium such as mineral or silicon oil. The MR damper is a semi-active control device that is capable of generating the magnitude of forces necessary for full-scale applications, while requiring only a battery for power (Dyke and Spencer, 1997). Additionally they have high yield strength that allows large force capacity and low viscosity. They exhibit stable hysteretic behavior over a wide temperature range.

In the literature the MR dampers' damping properties are regulated by different control strategies. Dyke and her co-workers performed acceleration feedback control strategies based on H_2/LQG methods (Dyke et al., 1996). Spencer and his co-workers tried frequency domain optimal control strategies by two specific techniques, the H_2 and H_∞ control methods (Spencer et al. 1994). Various research groups utilized fuzzy logic to control MR dampers (Choi et al., 2004) (Huang et al., 2009) (Schurter and Roschke, 2001) (Turan and Kinay, 2009) (Wilson, 2005). Dyke and Spencer compared semi-active control strategies for the MR damper and concluded that the performance of the control system is highly dependent on the choice of algorithm employed (Dyke and Spencer,

1997). Instantaneous optimal control with velocity and acceleration feedback was utilized and additionally the structural stability was guaranteeing by using the Lyapunov approach (Ribakov and Gluck, 2002). Sliding mode control was also applied to MR dampers (Kinay and Turan, 2009).

In the current research the controller consists of two stages: an optimal controller and a modified clipped algorithm. Yoshida and Dyke proposed the modified clipped control algorithm in which the control voltage lies between 0 and v_{max} (Yoshida and Dyke, 2004) (Yuen et al., 2007).

Safonov et al. (1981), Doyle et al. (1989), and Lu (2001) presented the framework for the LQG method, whereas Ramallo et al. (2002) applied an H_2/LQG control to a two storey model structure.

In the present research, the H_2/LQG control strategy is utilized due to the necessity of designing an observer. Additionally the linear quadratic regulator (LQR) method is employed in order to compare two different control strategies.

2. SEMI-ACTIVE MODIFIED CLIPPED OPTIMAL CONTROL

The block diagram of the semi-active modified clipped optimal control is given in Figure 1. The controller is composed of two stages. The first one is the optimal controller, and the second one is the modified clipped controller. The control signals that will cause the system to satisfy some physical constraints and at the same time maximize or minimize a chosen performance criteria (cost function) are determined by choosing the optimal control algorithm. Finally the magnetic field in the damper is set to develop damping forces that are equal to those obtained by the optimal control. This is performed by a modified clipped controller. The applied control voltage is limited in the range of $v \in [0, v_{max}]$. Hence the applied control force f_{MRD} is restricted.

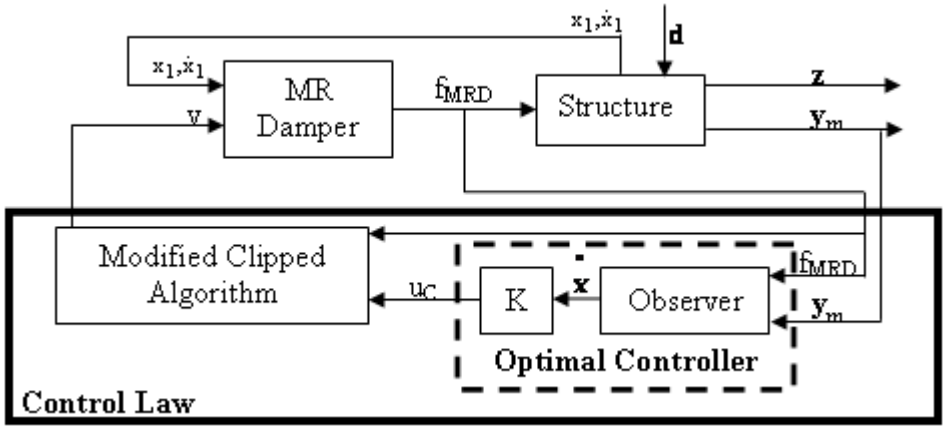

Figure 1. Semi-active control block diagram

The **z** vector is composed of the quantities which are aimed to be controlled. In the current application the real states **x** and the control input u_C are the **z** vector's elements. The $\mathbf{y}_m$ vector is composed of the measured quantities. The signal **d** contains all external inputs, including disturbances, and sensor noise.

2.1 H_2/LQG Control Algorithm

The H_2 optimal control theory has its roots in the frequency domain interpretation of the cost function associated with time-domain state-space LQG (linear quadratic Gaussian) control theory. The H_2 optimal controller F(s) is a combination of a Kalman filter with residual gain matrix K_f and a full-state feedback gain K_c which of both are determined in the usual linear quadratic regulator (LQR) manner. A stabilizing H_2 optimal linear time invariant

controller F(s) in state-space form is computed for a partitioned linear time invariant plant P(s). The controller F stabilizes the plant P and has the same number of states as P. The linear time invariant system P is partitioned where inputs to B_1 are the disturbances, inputs to B_2 are the control inputs, the second row give the errors to be kept small, and the output of the third row is the output measurement vector provided to the controller.

The signal **d** contains all external inputs, including disturbances, and sensor noise. The signal **z** is composed of the quantities which are aimed to be controlled. The H_2 norm of a transfer function matrix from **d** to **z**, $\mathbf{G}_{zd}$ is defined as

$$\|\mathbf{G}_{zd}\|_2 = \sqrt{\text{trace}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{G}_{zd}(j\omega)\mathbf{G}_{zd}^{*}(j\omega)\,d\omega\right]} \tag{1}$$

In order to perform a physical interpretation of the H_2 norm, one can note that the H_2 norm of a transfer function is equal to the RMS (root mean square) value of its output, **q** when the input is a unit white noise. The RMS output vector is defined by

$$\|\mathbf{q}\|_{rms} = \lim_{\tau\to\infty}\frac{1}{2\tau}\int_{-\tau}^{\tau}\mathbf{q}^{T}(t)\,\mathbf{q}(t)\,dt \tag{2}$$

This expression can also be written as

$$\|\mathbf{q}\|_{rms} = \sqrt{\sum_i E\left[q_i^2(t)\right]} \tag{3}$$

where E[.] indicates the expected value operator. After the current stage, the problem turns into a LQG (linear quadratic Gaussian) algorithm. The LQG optimal controller is simply composed of a Kalman filter (linear quadratic estimator, LQE) with a linear quadratic regulator (LQR). Two Riccati equations that come from the optimization of the performance indices are solved for the optimal solution. With appropriate selection of design weights, the H_2 optimal control criteria defined in the frequency domain can be numerically equivalent to the LQG optimal control criteria defined in the time domain (Lu, 2001).

2.2 Modified Clipped Algorithm

The required voltage to create the magnetic field around the damper is applied. Hence the MR damper provides the desired control forces. When the MR damper generate the desired control force u_C, the voltage should be kept at the present level ($u_C = f_{MRD}$). If the damper force's (f_{MRD}) magnitude is smaller than the desired control force's magnitude and both forces have the same sign, then maximum voltage should be applied in order to increase the MR damper force and to catch the desired control force level. Otherwise, zero voltage should be sent.

3. NUMERICAL SIMULATIONS

The seismic response of a three-storey model structure is simulated in order to compare the responses obtained by two different control strategies. The MR damper's response is modeled by a mechanical model which is proposed by Spencer et al. (1997) and Yang et al. (2002) to represent the danmper's nonlinear character. The MR damper is rigidly attached between the ground and the first floor of the structure. The equations of motion for the model structure is written by assuming that the controlled building response remains in the linear region.

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = -\boldsymbol{\Gamma} f_{MRD} - \boldsymbol{M}\boldsymbol{\Lambda}\ddot{x}_g \tag{4}$$

$$\boldsymbol{M} = \begin{bmatrix} 90 & 0 & 0 \\ 0 & 90 & 0 \\ 0 & 0 & 90 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 175 & -50 & 0 \\ -50 & 100 & -50 \\ 0 & -50 & 50 \end{bmatrix}, \boldsymbol{K} = 10^3 \begin{bmatrix} 18 & -10.26 & 0 \\ -10.26 & 20.55 & -10.26 \\ 0 & -10.26 & 10.26 \end{bmatrix} \tag{5}$$

where M, C, and K are the mass, damping, and stiffness matrices, respectively. The units are kg, Ns/m, and N/m respectively. x is the displacement vector. Dot indicates time derivative. f_{MRD} is the MR damper force. $\ddot{x}_g$ is the ground excitation. The location matrices of the control force and the ground excitation are as $\Gamma=[1\ 0\ 0]^T$ and $\Lambda=[1\ 1\ 1]^T$, respectively. The structure's periods are T=[1.43 0.49 0.33] seconds. The seismic data belong to the east-west component of the Bolu station record of the 1999 Düzce earthquake in Figure 2.

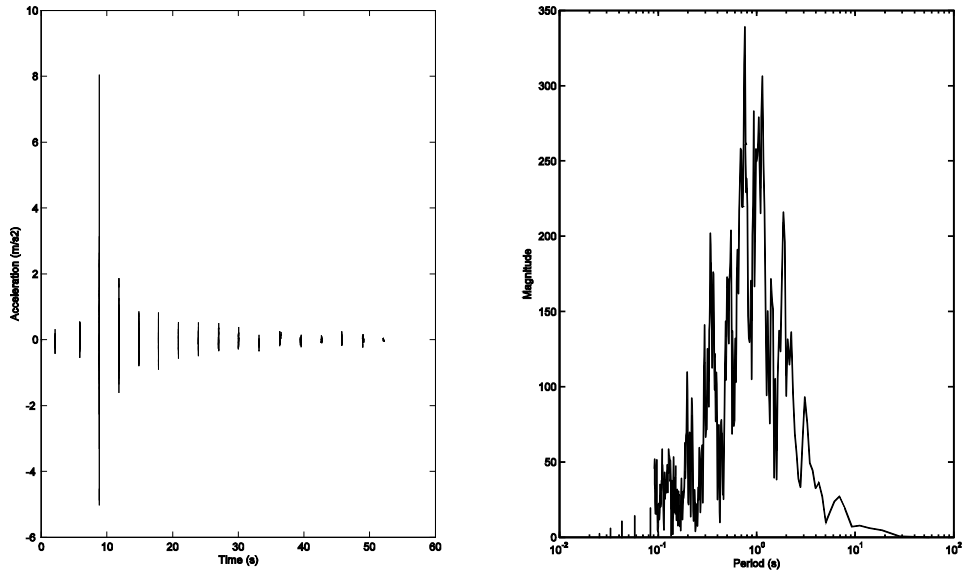

Figure 2. East-west component of the Bolu station record of the 1999 Düzce earthquake in time and frequency domains

For the H_2/LQG method, displacements are assumed to be sensed only. White noise with a magnitude of 5% of the maximum displacement values are added to the calculated responses as sensor noise. As part of this method, an observer is designed to obtain all the states. For the LQR method, on the other hand, since there is no observer involved in this case, all states need to be sensed. Again, white noise of 5% of the maximum values is added to the displacement and velocity signals.

The structure's interstory drift, interstory velocity, and absolute acceleration values for the first and third floors are presented in Figure 3 and Figure 4, respectively. As it can be seen from the graphs, the H_2/LQG control method is capable of effectively reducing both displacements, velocities, and absolute accelerations except one peak in the first floor's absolute acceleration. This case is an expected outcome due to the existence of the damper force in the first floor. On the other hand, the LQR control method produces some peaks that are not

present in the uncontrolled structure's velocity response. The LQR method's first floor absolute acceleration response is unacceptable due to some peaks close to 4g.

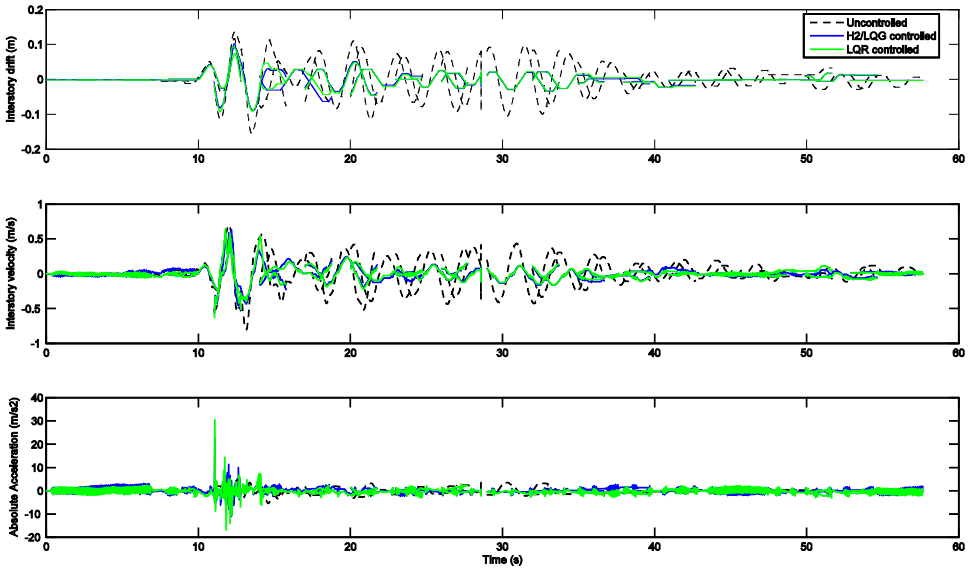

Figure 3. Interstory drift, interstory velocity, and absolute acceleration responses of the first floor

A 12.5 Hertz component in the first floor's response and in the damper's force seems to come from the dynamics of the MR damper. It is not filtered out intentionally by the authors in order to emphasize on the comparison of the results of two control methods, but not on the MR damper dynamics.

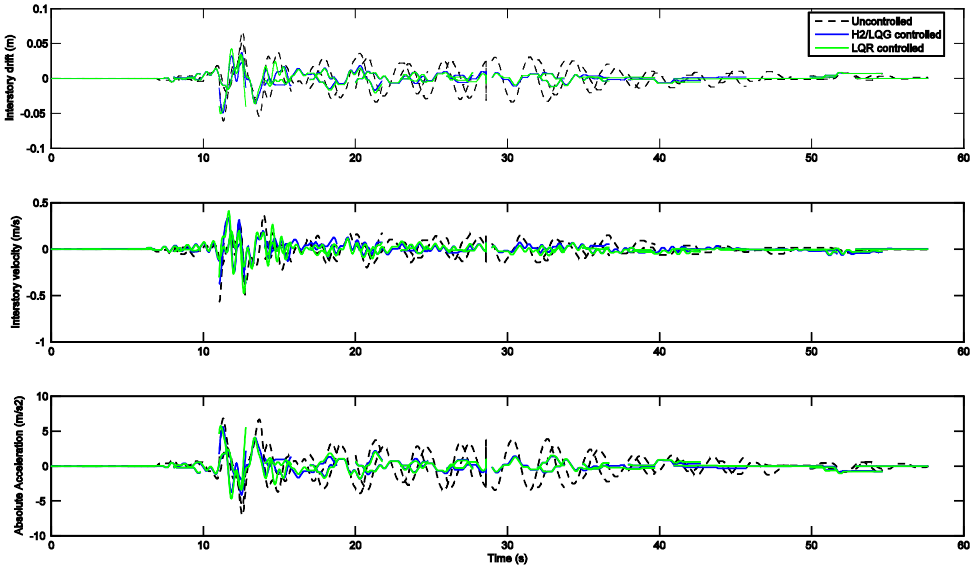

Figure 4. Interstory drift, interstory velocity, and absolute acceleration responses of the third floor

There exists an upper limit for the MR damper force due to the physical constraints. In the present study, the upper limit of the applied control force is 3000 N. The calculated control force and the damper force are given in Figure 5 for two control methods. Some undesirable peaks occur in the LQR method's damper force. Since these peaks are made up of single points, the MR damper can not be successful to perform these aggressive peaks.

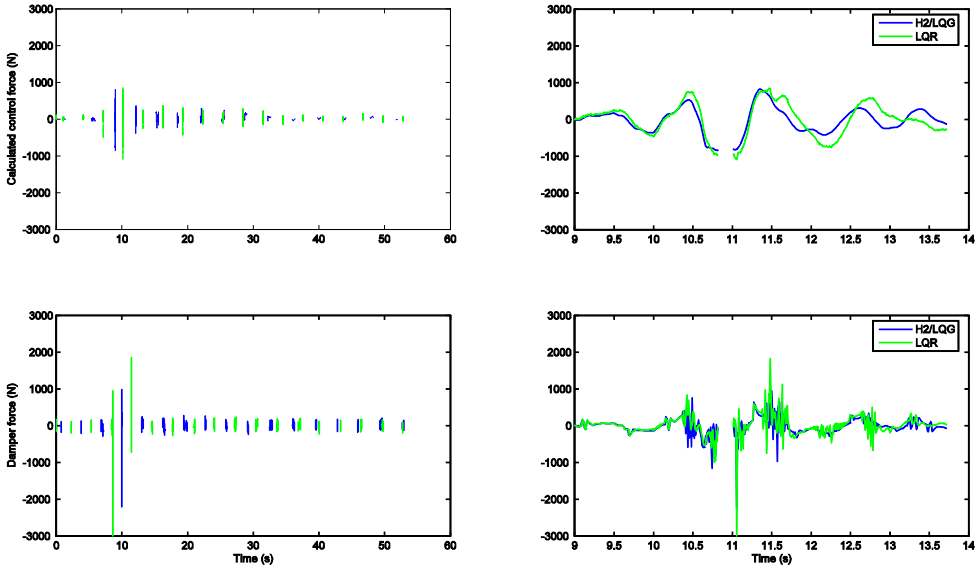

Figure 5. The calculated control force and the MR damper force (The responses between 9 and 14 seconds are zoomed in on the right column.)

The maximum and RMS values are given in Table 1. The H_2/LQG method's values are about 25% smaller than the LQR method's. Finally, one can conclude that the H_2/LQG controller performed better in terms of structural responses by a smaller amount of control force.

Table 1. The maximum and RMS values of the calculated control force and of the damper force for two methods

	H_2/LQG		**LQR**	
	MAX	RMS	MAX	RMS
u_C (N)	831	107	1118	174
f_{MRD} (N)	2237	94	3000	108

4. CONCLUSIONS

In the content of the current research, the performances of two control methods are investigated for a three-storey model structure to which an MR damper is attached. The results show that the H_2/LQG control reduces the structural responses by approximately 40%. The first floor's absolute acceleration values are slightly larger than the uncontrolled ones. This case is an expected outcome due to the existence of the MR damper that adds

force to that floor. Although the H_2/LQG method calculated smaller control forces than the LQR method, it performed a better control in terms of interstory drifts, interstory velocities, and absolute accelerations.

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