TRAVELING WAVE EFFECT OF EARTHQUAKE GROUND MOTION IN MULTI-STOREY BUILDINGS

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ABSTRACT:

Traveling waves may cause different vertical ground displacements affecting multi-story buildings which need to be analyzed by multi-support excitation. Raft foundations cause a diaphragm effect in the horizontal direction, and therefore multi-support excitation in the lateral direction is not considered. In this study, the effect of vertical earthquake motions onto high story buildings on elastic soil is investigated. Multi support excitation is considered by using displacement loading, which defines the equivalent seismic loads in terms of the ground displacement. A static correction procedure which is necessary for the modal analysis in displacement loading is presented. As a special case, only wave passage without reflection and refraction effect is considered and the results on the design forces are investigated.

KEYWORDS: Traveling Wave Effect, Multi Support Excitation, Displacement Loading, Static Correction.

1. INTRODUCTION

The earthquake response analysis of structures subjected to multi-support excitation analysis has gained the great interest in the earthquake engineering. Multi support excitation is a well-formed analysis tool that is widely used in earthquake response analysis of structures that have large footprints. Past research showed that the seismic wave passing through the ground may cause valuable change in the response of structures, if the supports are far apart. The change in the earthquake wave is not only considered with a time delay, but also should contain the path attenuation, reflection and refraction effects. However, if the supports of the structure are adequately close to each other, these effects may be neglected (Soyluk 2004, Li et al. 2012). Cui and Gao (2011) have investigated the traveling wave effect in the long-span cable stayed bridges, and they concluded that long-span cable-stayed bridges not only need to consider traveling wave effect, but also study on refraction, reflection and scattering of the waves in different medium of the underlying soil. Jihong et al. (2012) presented a simplified method to estimate multi-support excitation responses. In his study, the multi-support response spectrum was constructed by modification and extension of the existing response spectrum method under uniform excitation. Hızal and Turan (2016) have investigated the seismic analysis of a cable stayed bridge subjected to different support displacements and found out that the traveling wave results in a pure time delay in the base shear force response.

In the literature, only a few studies have been observed that deal with traveling wave effect in multi-story frames. Rambabu and Allam investigated the effect of apparent wave velocity in open frame structures with soil structure interaction. In their study, only the horizontal component of the ground motion was considered, and they observed a change in the dynamic response with small time delays in the ground acceleration. This observation may be reliable for the structures whose supports are able to move independently from each other. In many cases, the
multiple support excitation procedure is not possible to implement in high-rise buildings since they are built on raft foundation in which the supports are not able to move independently. However, the time delay in the vertical ground motion may cause a valuable change in the dynamic response of the structure since the foundation beam/or slab cannot be assumed infinitely rigid in the vertical direction.

In the multi-support analysis, pseudo-static displacement effects have an important role on the dynamic responses. For this reason, a transformation is required between the relative and absolute dynamic displacements. The displacement loading which defines the general equation of motion as depending on the absolute coordinates can be referred as more practical by comparison with the conventional acceleration loading. However, as a natural result of its definition, the displacement loading excites higher mode of vibration and requires nearly all modes in order to have a reliable dynamic response. Wilson (1998) summarized the general modal analysis procedure of displacement loading and proposed a static correction method for the consideration of higher mode effects. Hızal and Turan (2017) pointed out pseudo static effects in the base shear force response of cable stayed bridges and comprehended on the physical meaning of the static correction method for modal analysis.

In this study, the traveling wave effect of vertical ground motion in high-rise structures with soil structure interaction is investigated by using ground displacement loading. The effect of the variations in the apparent wave velocity on the element dynamic forces are investigated by using the finite element model of a twenty-story structure.

2. STATEMENT OF THE PROBLEM

The mathematical model of a soil structure interaction model is represented in Fig. (1). The foundation of the structure is incorporated by Winkler springs whose stiffness is equivalent to the soil stiffness.

![Figure 1. Mathematical model of multi-support soil-structure interaction system](image-url)
In the finite element formulation of a structure the soil structure interaction, the stiffness, mass and damping matrices must contain the structural information of both superstructure and foundation components. Therefore, the stiffness, mass and damping matrices of unconstrained (structural) degrees of freedom are obtained as

\[
K_u = \begin{bmatrix} K_s & K_{sf} \\ K_{sf} & K_{ff} \end{bmatrix}, \quad M_u = \begin{bmatrix} M_s & 0 \\ 0 & M_{ff} \end{bmatrix}, \quad C_u = \begin{bmatrix} C_s & C_{sf} \\ C_{sf} & C_{ff} \end{bmatrix}
\]  

where subscript \(K\), \(M\) and \(C\) are the stiffness, mass and damping matrices of superstructure, \(K_{sf}, M_{sf},\) and \(C_{sf}\) are the stiffness, mass, and damping matrices of the foundation, \(K_{fs}, M_{fs},\) and \(C_{fs}\) are the coupled stiffness, mass and damping matrices of the superstructure and foundation, respectively. Note that the matrices \(M_{sf}, C_{sf}\) and \(C_{fs}\) equal to zero if the mass and damping of the foundation can be neglected.

The equation of motion of the system presented in Fig. (1) is written as

\[
\begin{bmatrix} M_{uu} & 0 \\ 0 & M_{cc} \end{bmatrix} + \begin{bmatrix} \ddot{y}_u(t) \\ \ddot{y}_c(t) \end{bmatrix} + \begin{bmatrix} [C_{uu}] & [C_{uc}] \\ [C_{cu}] & [C_{cc}] \end{bmatrix} \begin{bmatrix} \dot{y}_u(t) \\ \dot{y}_c(t) \end{bmatrix} + \begin{bmatrix} K_{uu} & K_{uc} \\ K_{cu} & K_{cc} \end{bmatrix} \begin{bmatrix} y_u(t) \\ y_c(t) \end{bmatrix} = \begin{bmatrix} 0 \\ F_c(t) \end{bmatrix}
\]  

in which \(M_{uu}, C_{uu} \) and \(K_{uu}\) are the mass, stiffness and damping matrices of the constrained DOF, \(C_{uc} \) and \(K_{uc}\) denote the coupled damping and stiffness matrices. In addition, \(y_u(t)\) and \(y_c(t)\) are the absolute displacement vector of the unconstrained and constrained degrees of freedom. At the right-hand side of Eq. (2), \(F_c(t)\) denotes the equivalent external dynamic force acting on the constrained DOF. This force is considered to be the action force of the ground, that activates the constrained DOF.

The first and second row of Eq. (2) gives the equation of motion of the unconstrained and constrained degrees of freedom, respectively.

\[
M_{uu} \ddot{y}_u(t) + C_{uu} \dot{y}_u(t) + K_{uu} y_u(t) = -K_{uc} y_c(t)
\]

\[
M_{cc} \ddot{u}_c(t) + C_{cc} \dot{u}_c(t) + K_{cc} y_c(t) = F_c(t)
\]

The equivalent elastic force vector acting on the unconstrained DOF is obtained by

\[
f_u(t) = K_{uu} y_u(t) + K_{uc} y_c(t)
\]

whose equivalent is

\[
f_u(t) = K_{uu} v(t)
\]

in which \(v(t)\) denotes the relative displacement vector of the unconstrained DOF with respect to the constrained DOF. Thus, the following relation can be constructed between the displacement of constrained and unconstrained DOF by equating Eq. (6) and (5).

\[
y_u(t) = v(t) + R y_c(t)
\]
Here $R$ denotes the influence matrix with a size of $N \times N_c$ (Chopra 2013) and can be evaluated as

$$ R = -K_{uu}^{-1} K_{uc} $$

where $N$ and $N_c$ denote the number of unconstrained and constrained degrees of freedom, respectively.

3. MODAL ANALYSIS PROCEDURE

In the modal analysis, the displacement vector of the unconstrained degrees of freedom can be written as

$$ y_u(t) = \sum_{i=1}^{N} \phi_i(t) q_i(t) $$

where $\phi_i$ and $q_i(t)$ are the modal shape vector and normal coordinate function of $i^{th}$ mode. The uncoupled equation of motion of the $i^{th}$ mode can be obtained by substituting Eq. (9) into the Eq. (3) and multiplying by the $\phi_i^T$ in the right-hand side as

$$ M_i \ddot{q}_i(t) + C_i \dot{q}_i(t) + K_i q_i(t) = -\phi_i^T K_{uc} y_u(t) $$

Where $M_i$, $C_i$ and $K_i$ denotes the generalized mass, damping and stiffness matrix of the $i^{th}$ mode, respectively.

It can be seen the equivalent load vector given at the right-hand side of Eq. (10) is proportional to ground displacement and referred as displacement loading in the literature. If the both side of the Eq. (10) is divided by $M_i$, the uncoupled equation of motion will turn into the following form:

$$ \ddot{q}_i(t) + 2\zeta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = \Gamma_i^k y_c(t) $$

in which $\zeta_i$, $\omega_i$ and $\Gamma_i^k$ indicates the damping ratio, natural angular frequency and modal participation factor of the $i^{th}$ mode.

$$ \Gamma_i^k = \frac{-\phi_i^T K_{uc}}{\phi_i^T M_{uu} \phi_i} $$

Note that the modal participation factor given in the Eq. (13) is associated with ground displacement vector. If the Eq. (8) is substituting into the right-hand side of Eq. (13), the following equation will be obtained.

$$ \Gamma_i^k = \frac{-\phi_i^T K_{uc} R}{\phi_i^T M_{uu} \phi_i} $$
Eq. (14) turns into following form if the $K_{uu}$ is substituted by $-\omega_i^2 M_{uu}$.

$$
\Gamma_i = \Gamma_i^m \omega_i \tag{15}
$$

Where $\Gamma_i^m$ denotes the modal participation factor obtained for acceleration loading.

$$
\Gamma_i^m = \frac{\phi_i^T M_{uu} R}{\phi_i^T M_{uu} \phi_i} \tag{16}
$$

As it is known from the literature, the displacement loading excites higher modes of vibration (Tsai 1998, Wilson 2002). This reason of this case lays on the physical mean of the Eq. (15). In the conventional acceleration loading in which the equation of motion is constructed due to the relative displacements, the modal participation decreases in the higher modes. This case also causes a decreasing in the higher mode response. However, in the displacement loading an increase is occurred in the modal participation factor due to the multiplier of $-\omega_i^2$. Consequently, the response of the higher modes will be dominant in the total response. In order to overcome this problem, a static correction procedure can be applied. In the static correction procedure, the higher mode response is taken in to account by using the pseudo static component of the absolute displacements.

In the Eq. (3), the modal expansion of the ground displacement can be written as

$$
K_{uu} = -\sum_{i=1}^{N} M_{uu} \phi_i \Gamma_i \tag{17}
$$

Substituting the $M_{uu}$ with $-\omega_i^2 K_{uu}$ and after arrangements

$$
R = \sum_{i=1}^{N} \frac{\phi_i \Gamma_i}{\omega_i^2} \tag{18}
$$

Thus, the higher mode response is represented by its pseudo-static component as

$$
y_p(t) = \left[ R - \sum_{i=1}^{N} \frac{\phi_i \Gamma_i}{\omega_i^2} \right] y_i(t) \tag{19}
$$

where $n$ denotes the number of considered modes. Finally, the corrected absolute displacement response is obtained as

$$
y_a(t) = \sum_{i=1}^{n} \phi_i q_m(t) + y_p(t) \tag{20}
$$

4. NUMERICAL ANALYSIS

A twenty story, two-bay moment frame on a 1m thick raft foundation is investigated. Elastic soil stiffness is considered as $C_z = 10,000,000$ KN/m in the vertical direction and $C_x = 100,000,000$ KN/m in the horizontal direction. In the analysis, two-dimensional finite element model of the strip shown in Fig. (2) is constructed. The
considered strip of the raft foundation is divided into bar elements with a length of 1.00 m. The vertical and horizontal component of the El Centro 1940 ground motion is considered in the analysis [see Fig (3)]. Note that the horizontal component causes a uniform ground motion while the vertical component results in multiple support excitations due to the time delay.

![Plan view of the 20 story, 7 bay moment frame](image)

**Figure 2.** Plan view of the 20 story, 7 bay moment frame

<table>
<thead>
<tr>
<th>Number of stories ($N_s$)</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Story height</td>
<td>300 cm</td>
</tr>
<tr>
<td>Columns</td>
<td>75 x 75 cm</td>
</tr>
<tr>
<td>Beams</td>
<td>30 x 70 cm</td>
</tr>
<tr>
<td>Slab thickness</td>
<td>15 cm</td>
</tr>
<tr>
<td>Foundation thickness</td>
<td>100 cm</td>
</tr>
<tr>
<td>$E_C$</td>
<td>30,000 KN/m$^2$</td>
</tr>
<tr>
<td>$C_X$</td>
<td>100,000,000 KN/m</td>
</tr>
<tr>
<td>$C_Z$</td>
<td>10,000,000 KN/m$^2$</td>
</tr>
</tbody>
</table>

![Horizontal component of El Centro 1940 ground motion](image)

**Figure 3.** Horizontal and vertical components of El Centro 1940 ground motion
In Figs. (4-6), the variations in the normalized bending moment, shear and axial force of the columns S1, S3, S5 and S7 with respect to the traveling wave velocity, $V_s$, are presented. The normalized internal forces are the ratio of the calculated internal force to its maximum value based on uniform ground excitation. At first view, it can be observed that the normalized bending moment and shear forces asymptotically approach their maximum value versus the wave velocity. In addition, a 4-6 % peak difference is observed for $V_s = 200$ m/sec. in S1, S3, and S5, respectively. Axial forces, S1 and S3 follow the same trend as mentioned above, but S5 and S7 reach their maximum values in the range of $V_s = 150-250$ m/sec.

Figure 4. Variations in normalized bending moment versus traveling wave velocity

Figure 5. Variations in normalized shear force versus traveling wave velocity

Figure 6. Variations in normalized axial force versus traveling wave velocity
5. CONCLUSIONS

In this study, the traveling wave effect of vertical ground motion in multi-story structures is investigated with soil structure interaction. The model is constructed by using Winkler springs and multi-support excitation is considered in the dynamic analysis. The dynamic equilibrium equation is constructed based on the ground displacement and absolute displacement vector. A static correction procedure which is necessary for the dynamic analysis is introduced to reduce the required number of modes in the modal analysis procedure.

In the numerical analysis, a twenty-story reinforced concrete moment frame on elastic foundation is considered. The vertical and horizontal components of the El Centro 1940 ground motion is defined to the constrained degrees of freedom and the seismic wave velocity is considered between 100 and 1000 m/sec. In the analysis results, it is seen that the maximum normal force response of the first story interior columns show a change from -25% to 12% due to the variations in seismic wave velocity. On the other hand, a small change of 4-6% is observed in bending moment and shear force responses.

REFERENCES


