LINEAR AND NONLINEAR SITE RESPONSE ANALYSIS BY USING DISCRETE TIME FILTERS

R. Şişman¹, E. Şafak² and A. Şahin³

¹ Res. Assist., Civil Eng. Department, Yıldız Technical University, Istanbul
² Prof. Dr., Kandilli Observatory and Earthquake Research Institute, Bogazici University, Istanbul
³ Assoc. Prof. Dr., Civil Eng. Department, Yıldız Technical University, Istanbul
Email: rafet@yildiz.edu.tr

ABSTRACT:

We present MATLAB/SIMULINK-based tools for linear and nonlinear site response analysis by using discrete-time filters. Site response analysis methods that are given in the literature are mainly in frequency domain, and consequently the nonlinearity is handled by equivalent linear techniques. In this study, we present a time-domain approach by modeling the wave propagation in soil media by time-varying discrete-time filters. A Simulink model is developed for the calculations. Simulink is a MATLAB-based digital simulation environment to create analytical models by simply connecting mathematical operation blocks. In this study, we utilized discrete-time filter blocks to create the model for site response analysis. The nonlinearity in the model is accounted for by calculating at each time step the soil strains and adjusting the wave propagation velocity accordingly. We test the accuracy of the proposed approach on numerical models by comparing the results with those of the frequency-domain equivalent linear techniques.

KEYWORDS: Site Response Analysis, Nonlinear Analysis, Simulink, Discrete Time Filters, Wave Propagation

1. INTRODUCTION

During earthquakes, one of the important factors affecting the shaking intensity on ground surface is site amplification. Site amplification can be measured by field tests, or calculated by using analytical approaches. There are a large number of methods to calculate site response analytically (Idriss and Seed, 1968; Roësset et al., 1969; Borcherdt, 1994; Schnabel et al., 1972). Soil site is generally modelled as a continuous medium, which typically requires frequency domain analysis. An alternative approach is to consider soil layers as a concentrated mass-stiffness system, and analyze it in the time domain. Nonlinear site response can be calculated approximately by utilizing equivalent linear analysis in the frequency domain. For more accurate nonlinear site response, time domain analysis is required. This study presents a time-domain wave propagation approach to calculate site response for a continuous soil medium. The MATLAB-based Simulink (MATLAB 2014a) software is utilized to formulate and solve the problem in time domain by using discrete-time filters.

2. FORMULATION

For a soil site with m layers, and using the notation given in Figure 1, we can write the following equations for the up-going and down-going shear wave amplitudes (Şafak, 1995):

\[ u_j(t) = R_{d,j-1}d_j(t - \tau_j) + T_{u,j-1}u_{j-1}(t - \tau_j) \]  \hspace{1cm} (1.a)

\[ d_j(t) = T_{d,j}d_{j+1}(t - \tau_j) + R_{u,j}u_j(t - \tau_j) \]  \hspace{1cm} (1.b)
In the equations above $u_j(t)$ and $d_j(t)$ refers to the up-going and down-going wave amplitudes at time $t$ at the top and the bottom of layer $j$, respectively, and $j = 1, ..., m$. $R_{u,j}, T_{u,j}$ and $R_{d,j}, T_{d,j}$ are the reflection and transmission coefficients for the up-going and down-going waves at interface $j$. By using up-going and down-going waves, one can calculate amplitudes of vibration at any interface as:

$$y_j(t) = u_j(t) + d_j(t - \tau_j)$$ (2)

In Equation 2, $y_j(t)$ can be accelerations, velocities or displacements.

The formulation given up to this point shows the recursive equations required for undamped wave propagation in layered soil media. In Şafak (1995), the quality factor approach (Knopoff, 1967) is used to incorporate damping into the equations. Damping decay function for a soil layer with travel time $\tau$ and quality factor $Q$ is given as:

$$A(f) = e^{-\pi f/\bar{Q}}$$ (3)

Here, the decay function is frequency dependent and needs to be converted into discrete time domain in order to continue calculations in time domain. The attenuation due to damping can be approximated by a discrete time filter of the following form Şafak (1995):

$$\lambda = \frac{1 - \alpha}{2} \cdot \frac{1 + q^{-1}}{1 - \alpha q^{-1}} \quad \alpha = \frac{1 - \sqrt{1 - \cos^2 \theta}}{\cos \theta} \quad \theta = \ln 2 \frac{Q\Delta}{\tau} \leq \pi$$ (4)

In Equation 4, $\Delta$ refers to sampling time and $q^{-1}$ is unit time shift (i.e. unit lag operator). As stated in the equation, in order to have a stable filter, $\theta$ value should be less than $\pi$, which requires that the sampling interval $\Delta$ should be small enough to satisfy this criterion. Damped motion amplitudes can be calculated by using Equation 5, where $\tilde{u}_j$ and $\tilde{d}_j$ denotes damped up-going and down-going wave amplitudes.

$$\tilde{u}_j(t) = \lambda u_j(t) \quad \tilde{d}_j(t) = \lambda d_j(t)$$ (5)

### 3. Designing Filters

To incorporate the damping filter in Eqs. 1, we first re-write the equations by using the time shift operator $q$. Equation 1.a can be written as:

$$u_j(t) = R_{d,j-1}d_j(t)q^{-\tau_j} + T_{u,j-1}u_{j-1}(t)q^{-\tau_j}$$ (6)
Incorporating damping filter defined in Equation 4 into Equation 6 yields:

\[
\ddot{u}_j(t) = \frac{1 - \alpha}{2} \cdot q^{-\tau_j} + q^{-\tau_j - 1} \frac{1 - \alpha}{1 - \alpha q^{-1}} \cdot R_{d,j-1} d_j(t) + \frac{1 - \alpha}{2} \cdot q^{-\tau_j} + q^{-\tau_j - 1} \frac{1 - \alpha}{1 - \alpha q^{-1}} \cdot T_{u,j-1} u_{j-1}(t)
\]

(7)

Or, in a more compact form:

\[
\ddot{u}_j(t) = \mathcal{F}_{d,j}(q) \cdot d_j(t) + \mathcal{F}_{u,j-1}(q) \cdot u_{j-1}(t)
\]

(8)

Where \(\mathcal{F}_{d,j}(q)\) and \(\mathcal{F}_{u,j-1}(q)\) are defined as:

\[
\mathcal{F}_{d,j}(q) = \frac{1 - \alpha}{2} \cdot q^{-\tau_j} + q^{-\tau_j - 1} \frac{1 - \alpha}{1 - \alpha q^{-1}} \cdot R_{d,j-1}
\]

(9.a)

\[
\mathcal{F}_{u,j-1}(q) = \frac{1 - \alpha}{2} \cdot q^{-\tau_j} + q^{-\tau_j - 1} \frac{1 - \alpha}{1 - \alpha q^{-1}} \cdot T_{u,j-1}
\]

(9.b)

Similarly, expressions can be derived for Equation 1.b.

Simulink software provides a large number of signal processing tools. Among these tools there are time delay blocks, FIR filter blocks and IIR filter blocks. Soil site response problems can be defined by using these blocks in accordance with the formulation given above.

For non-linear case, all the terms in the filters must be time-varying. Equations below show the corresponding filters with time-varying parameters.

\[
\mathcal{F}_{d,j}(q,t) = \frac{1 - \alpha(t)}{2} \cdot q^{-\tau_j(t)} + q^{-\tau_j(t) - 1} \frac{1 - \alpha(t)}{1 - \alpha(t) q^{-1}} \cdot R_{d,j-1}(t)
\]

(10.a)

\[
\mathcal{F}_{u,j-1}(q,t) = \frac{1 - \alpha(t)}{2} \cdot q^{-\tau_j(t)} + q^{-\tau_j(t) - 1} \frac{1 - \alpha(t)}{1 - \alpha(t) q^{-1}} \cdot T_{u,j-1}(t)
\]

(10.b)

4. NUMERICAL EXAMPLE FOR LINEAR CASE

We have developed Simulink software to perform the calculations formulated above. Simulink block diagrams were also utilized in some recent studies to perform site response analysis (Şafak, 2004; Şahin, 2015a, 2015b). However, in these studies discrete time filter blocks of Simulink are not directly used.

In the numerical examples for linear and nonlinear soil response, we used the same soil site. We assumed a single soil layer with a thickness of 25 m, initial shear modulus of 3.75e5 kPa, unit mass of 1500 kg/m³ lying over a bedrock with shear modulus of 2e6 kPa and unit mass of 2000 kg/m³. Initial damping ratio for the soil layer is assumed to be 1% which corresponds to quality factor of Q=50.

We considered two input motions, a 10 Hz sinusoidal signal, and an acceleration record from the 1995 Kobe, Japan earthquake (PEER NGA West Database). Input time histories are illustrated in Figure 2.a and Figure 2.b, respectively, and a schematic sketch of soil site is represented in Figure 3.
Figure 2.a – 10 Hz sinusoidal signal time history

Figure 2.b – Acceleration time history of EW component of Kobe 1995 earthquake

Figure 3 – Schematic sketch of the soil site used in examples

We have compared the results calculated from this methodology with those of Deepsoil (Hashash et al., 2016), a commonly used soil analysis software. Figure 4 shows the comparison of accelerations at the ground surface for the 10 Hz sinusoidal input, whereas Figure 5 shows the ground surface displacements for the Kobe earthquake input.
For 10 Hz sinusoid there are some differences. It looks like Deepsoil (Hashash et al., 2016) cannot account for the high frequency reflections generated by high frequency excitation. The match is much better for the earthquake excitation.

5. NUMERICAL EXAMPLE FOR NONLINEAR CASE

A well-known nonlinear soil model (Vucetic and Dobry, 1991) is used to account for the changes in stiffness and damping with soil strain. They are shown in Figure 6.

We have shown the results for the earthquake excitation only. In order to have stronger soil nonlinearity, we have multiplied the acceleration amplitudes by 2.

To account for soil nonlinearity, we have checked the soil strain at every time step and adjusted the filter parameters (i.e., velocities, reflection and transmission coefficients, and the parameters of damping filter) accordingly. We assumed no plastic deformations, and the same stiffness for loading and unloading curves at any strain level.

Figure 7 shows the comparison of the calculated ground surface displacements with those calculated using Deepsoil (Hashash et al., 2016). Although the general pattern is very similar, the peak displacements differ significantly; 63.7 cm with Deepsoil (Hashash et al., 2016), 154.5 cm with discrete-time filtering approach. We think that the approximations made in Deepsoil (Hashash et al., 2016) is the cause of the differences.
6. CONCLUSIONS

We present a discrete-time filtering approach to calculate linear and nonlinear soil response. The approach is based on Şafak (1995), and utilizes wave propagation formulation in layered media. Nonlinear soil response is incorporated in the formulation by using filters with time varying parameters. Time variations are accounted for by updating the filter parameters at every time step, compatible with the given nonlinear soil model.

A MATLAB-based Simulink (MATLAB 2014a) model is developed to perform the calculations by using its library of discrete time filter blocks.

Three numerical examples are presented to show the methodology, and the results are compared to those of Deepsoil (Hashash et al., 2016), a commonly used soil analysis software. The methodology presented gives more accurate soil response, especially at high frequencies and nonlinearities.

7. REFERENCES


PEER Ground Motion Database, http://ngawest2.berkeley.edu/.
